MARKOV'S MODEL OF MEANS OF TRANSPORT OPERATION AND MAINTENANCE IN A CITY TRANSPORT SYSTEM

M. Woropay, K. Migawa, A. Wdzięczny

University of Technology and Life Sciences
Department of Machine Maintenance
Prof. S. Kaliskiego Street 7, 85-789 Bydgoszcz, Poland
e-mail: kem@utp.edu.pl

Abstract

The paper deals with an analysis and mathematical model of means of transport operation and maintenance. The considerations are based on a selected real transport means system – city bus transport system in a chosen agglomeration.

The model has been built on the basis of a real city transport system involving realization of the process of operation and maintenance of means of transport. For this purpose, the process significant states and possible transitions between theses states have been determined. On this basis, an event model of means of transport operational use process was built, and next its mathematical model was made, with the assumption of its being the homogenous Markov's model.

Means of transport values of boundary probabilities for their being in the process particular states, were determined for data concerning their operational use. This serves as the basis for an analysis of the studied process of operational use.

The presented Markov's model of the means of transport operational use process is the effect of accomplishment of the first stage of the resultant model (semi-Markov's model) of operational use. The resultant model will be a part of a wide scale, decision model for creation and assessment of the transport system being ready for operation.

Keywords: *city transport system, operation and maintenance process, Markov's model*

1. Introduction

Transport system can be defined as a system used for satisfying transport needs, being realized along assigned routes. Operational use of transport means is a controlled process and can roughly be divided into the process of their operation and maintenance. The operation process involves realization of the transport means system basic task, thereby generation of profits. The maintenance process aims at maintaining the used technical objects in the state of being ready for operation.

In order to ensure high efficiency of the transport system operation it is necessary to take rational control decisions' concerning the transport means operational use process and, on the basis of its analysis, properly assess the process realization. The analysis and assessment of controlled processes is carried out by testing their models [6, 8]. Due to random character of the factors determining efficiency and speed of the process realized within a complex process of operational use, these are the stochastic processes which are most frequently applied for the process mathematical modelling. Among random processes, Markov and semi-Markov's ones have found a wide application for modelling the technical object exploitation process [3, 5]. Accomplishment of modelling research with the use of the described models of operational use, enables both the analysis of detailed problems connected with technical objects operation and maintenance, and of relations occurring between a determined number of the model parameters. Markov and semi-Markov's processes have been discussed explicitly in literature by many authors in a theoretical way [1, 2, 4, 7]. However, there are few examples of applications of these models for the description of the operational use processes realized in real transport systems, especially in city transport ones.

2. Event model of the operational use process

The operational use process model was built on the basis of analysis of the space of states and events concerning technical objects (city buses) used in the analyzed real transport system. In result of identification of the analyzed transport system, there has been determined multi-state process of technical object operation and maintenance realized within it, significant states of the exploitation process and possible transitions between these states. On this basis a graph of the operational use process state changes, has been built and presented in Fig. 1.

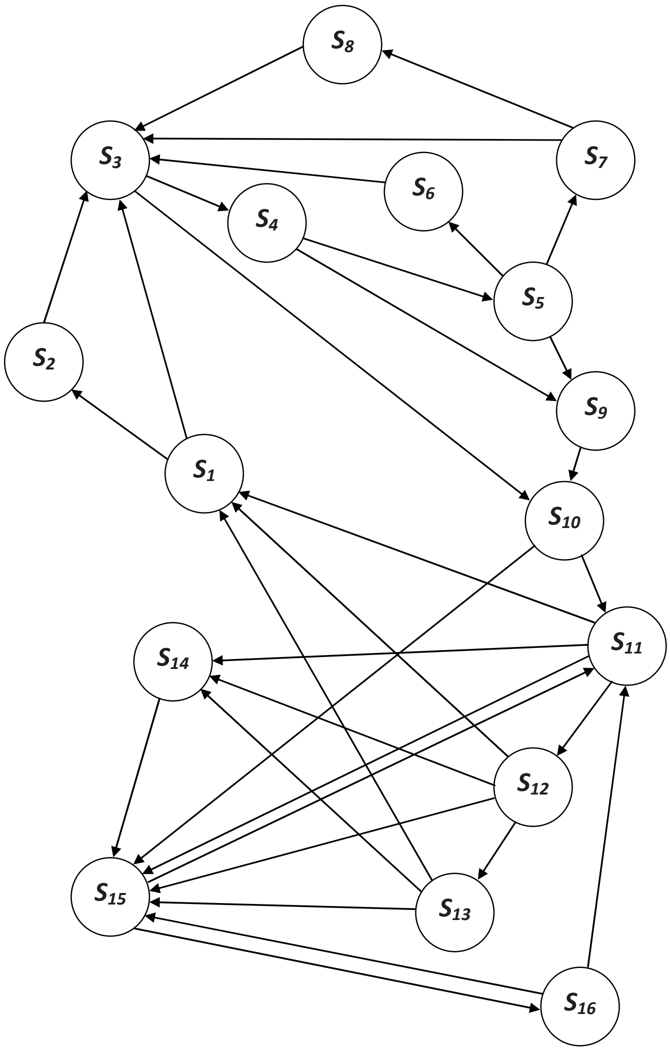

Fig. 1. Directed graph representing means of transport operation and maintenance process

Below, there have been presented names of the operational use states for the process of operation and maintenance of city bus transport means (Fig. 1):

S_1 - waiting for the task to be performed, on the hardstand of the depot (initial state),
S_2 - preparing for operation on the hardstand of the depot,
S_3 - carrying out the transport task,
S_4 - waiting for the decision of the traffic controller after occurrence of the vehicle damage,
S_5 - diagnosing by the emergency service unit,
S_6 - repair by the emergency service without losing the ride,
S_7 - repair by the emergency service with losing a ride,
S_8 - waiting for the task to be performed after repair by the emergency service,
S_9 - emergency exit,

S_{10} - waiting for action of the maintenance subsystem,
S_{11} - refuelling,
S_{12} - maintenance check on the operation day,
S_{13} - realization of periodical servicing,
S_{14} - prior to repair diagnosing in the serviceability assurance subsystem,
S_{15} - repair in the serviceability assurance subsystem,
S_{16} - diagnosing after the repair in the serviceability assurance subsystem.

3. Mathematical model of the operational use process

In result of carried out analysis of assumptions and restrictions, Markov's process *X(t)* was assumed to be the model of a technical object operational use. Using Markov's process for the operational use process mathematical modelling, the following assumptions have been accepted:

- Markov's process *X(t)* reflects the modelled real process properly enough from the point of view of the tests,
- the modelled process of operation and maintenance has a finite number of states S_i, $i = 1, 2, \ldots, 16$,
- random process *X(t)* which is a mathematical model of operational use is a homogenous process,
- in time $t = 0$ the process is in state S_1 (S_1 is the initial state).

On the basis of directed graph, presented in Fig. 1, there was built a matrix of P states change probabilities and matrix of Λ states change intensity of *X(t)* process:

$$P=\begin{bmatrix}
0 & p_{1,2} & p_{1,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & p_{2,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & p_{3,4} & 0 & 0 & 0 & 0 & 0 & p_{3,10} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & p_{4,5} & 0 & 0 & 0 & p_{4,9} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & p_{5,6} & p_{5,7} & 0 & p_{5,9} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & p_{6,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & p_{7,3} & 0 & 0 & 0 & 0 & p_{7,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & p_{8,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{9,10} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{10,11} & 0 & 0 & 0 & p_{10,15} & 0\\
p_{11,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{11,12} & 0 & p_{11,14} & p_{11,15} & 0\\
p_{12,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{12,13} & p_{12,14} & p_{12,15} & 0\\
p_{13,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{13,14} & p_{13,15} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{14,15} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{15,11} & 0 & 0 & 0 & 0 & p_{15,16}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{16,11} & 0 & 0 & 0 & p_{16,15} & 0
\end{bmatrix}, \qquad (1)$$

$$\Lambda=\begin{bmatrix}
-\lambda_{1,1} & \lambda_{1,2} & \lambda_{1,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -\lambda_{2,2} & \lambda_{2,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -\lambda_{3,3} & \lambda_{3,4} & 0 & 0 & 0 & 0 & 0 & \lambda_{3,10} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -\lambda_{4,4} & \lambda_{4,5} & 0 & 0 & 0 & \lambda_{4,9} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & -\lambda_{5,5} & \lambda_{5,6} & \lambda_{5,7} & 0 & \lambda_{5,9} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & \lambda_{6,3} & 0 & 0 & -\lambda_{6,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & \lambda_{7,3} & 0 & 0 & 0 & -\lambda_{7,7} & \lambda_{7,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & \lambda_{8,3} & 0 & 0 & 0 & 0 & -\lambda_{8,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_{9,9} & \lambda_{9,10} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_{10,10} & \lambda_{10,11} & 0 & 0 & 0 & \lambda_{10,15} & 0\\
\lambda_{11,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_{11,11} & \lambda_{11,12} & 0 & \lambda_{11,14} & \lambda_{11,15} & 0\\
\lambda_{12,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_{12,12} & \lambda_{12,13} & \lambda_{12,14} & \lambda_{12,15} & 0\\
\lambda_{13,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_{13,13} & \lambda_{13,14} & \lambda_{13,15} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_{14,14} & \lambda_{14,15} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{15,11} & 0 & 0 & 0 & -\lambda_{15,15} & \lambda_{15,16}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{16,11} & 0 & 0 & 0 & \lambda_{16,15} & -\lambda_{16,16}
\end{bmatrix}. \qquad (2)$$

where:
p_{ij} - probability of transition from state S_i to state S_j,
λ_i - intensity of staying in state S_i of $X(t)$ process,
λ_{ij} - intensity of transition from state S_i to state S_j.

On the basis of P (1) matrixes, in order to determine boundary probabilities p_i^* for Markov's chain, a system of linear equations was built

$$\sum_{i=1}^{9} p_{ij} \cdot p_i^* = p_j^*, \quad j = 1,2,...,9, \tag{3}$$

hence, according to (1):

$$\begin{cases} p_{11,1} \cdot p_{11}^* + p_{12,1} \cdot p_{12}^* + p_{13,1} \cdot p_{13}^* = p_1^*, \\ p_{1,2} \cdot p_1^* = p_2^*, \\ p_{1,3} \cdot p_1^* + p_{2,3} \cdot p_2^* + p_{6,3} \cdot p_6^* + p_{7,3} \cdot p_7^* + p_{8,3} \cdot p_8^* = p_3^*, \\ p_{3,4} \cdot p_3^* = p_4^*, \\ p_{4,5} \cdot p_4^* = p_5^*, \\ p_{5,6} \cdot p_5^* = p_6^*, \\ p_{5,7} \cdot p_5^* = p_7^*, \\ p_{7,8} \cdot p_7^* = p_8^*, \\ p_{4,9} \cdot p_4^* + p_{5,9} \cdot p_5^* = p_9^*, \\ p_{3,10} \cdot p_3^* + p_{9,10} \cdot p_9^* = p_{10}^*, \\ p_{10,11} \cdot p_{10}^* + p_{15,11} \cdot p_{15}^* + p_{16,11} \cdot p_{16}^* = p_{11}^*, \\ p_{11,12} \cdot p_{11}^* = p_{12}^*, \\ p_{12,13} \cdot p_{12}^* = p_{13}^*, \\ p_{11,14} \cdot p_{11}^* + p_{12,14} \cdot p_{12}^* + p_{13,14} \cdot p_{13}^* = p_{14}^*, \\ p_{10,15} \cdot p_{10}^* + p_{11,15} \cdot p_{11}^* + p_{12,15} \cdot p_{12}^* + p_{13,15} \cdot p_{13}^* + p_{14,15} \cdot p_{14}^* + p_{16,15} \cdot p_{16}^* = p_{15}^*, \\ p_{15,16} \cdot p_{15}^* = p_{16}^*, \end{cases} \tag{4}$$

was obtained. Whereas, in order to determine boundary probabilities p_i^* for Markov's process, a system of linear equations was built, on the basis of Λ (2) matrixes

$$\sum_{i=1}^{9} \lambda_{ij} \cdot p_i^* = 0, \quad j = 1,2,...,9, \tag{5}$$

hence, according to (2):

$$\begin{cases} -\lambda_{1,1} \cdot p_1^* + \lambda_{11,1} \cdot p_{11}^* + \lambda_{12,1} \cdot p_{12}^* + \lambda_{13,1} \cdot p_{13}^* = 0, \\ \lambda_{1,2} \cdot p_1^* - \lambda_{2,2} \cdot p_2^* = 0, \\ \lambda_{1,3} \cdot p_1^* + \lambda_{2,3} \cdot p_2^* - \lambda_{3,3} \cdot p_3^* + \lambda_{6,3} \cdot p_6^* + \lambda_{7,3} \cdot p_7^* + \lambda_{8,3} \cdot p_8^* = 0, \\ \lambda_{3,4} \cdot p_3^* - \lambda_{4,4} \cdot p_4^* = 0, \\ \lambda_{4,5} \cdot p_4^* - \lambda_{5,5} \cdot p_5^* = 0, \\ \lambda_{5,6} \cdot p_5^* - \lambda_{6,6} \cdot p_6^* = 0, \\ \lambda_{5,7} \cdot p_5^* - \lambda_{7,7} \cdot p_7^* = 0, \\ \lambda_{7,8} \cdot p_7^* - \lambda_{8,8} \cdot p_8^* = 0, \\ \lambda_{4,9} \cdot p_4^* + \lambda_{5,9} \cdot p_5^* - \lambda_{9,9} \cdot p_9^* = 0, \\ \lambda_{3,10} \cdot p_3^* + \lambda_{9,10} \cdot p_9^* - \lambda_{10,10} \cdot p_{10}^* = 0, \\ \lambda_{10,11} \cdot p_{10}^* - \lambda_{11,11} \cdot p_{11}^* + \lambda_{15,11} \cdot p_{15}^* + \lambda_{16,11} \cdot p_{16}^* = 0, \\ \lambda_{11,12} \cdot p_{11}^* - \lambda_{12,12} \cdot p_{12}^* = 0, \\ \lambda_{12,13} \cdot p_{12}^* - \lambda_{13,13} \cdot p_{13}^* = 0, \\ \lambda_{11,14} \cdot p_{11}^* + \lambda_{12,14} \cdot p_{12}^* + \lambda_{13,14} \cdot p_{13}^* - \lambda_{14,14} \cdot p_{14}^* = 0, \\ \lambda_{10,15} \cdot p_{10}^* + \lambda_{11,15} \cdot p_{11}^* + \lambda_{12,15} \cdot p_{12}^* + \lambda_{13,15} \cdot p_{13}^* + \lambda_{14,15} \cdot p_{14}^* - \lambda_{15,15} \cdot p_{15}^* + \lambda_{16,15} \cdot p_{16}^* = 0, \\ \lambda_{15,16} \cdot p_{15}^* - \lambda_{16,16} \cdot p_{16}^* = 0, \end{cases} \tag{6}$$

was obtained.

Analytical solution of linear equation systems (4) and (6), is theoretically possible, though time and work consuming, and the obtained formulas describing boundary probabilities p_i^* are complex and complicated, both for Markov's chain and process. For the purpose of determination of boundary probabilities p_i^*, a computer program enabling solution of equation systems (4) and (6) was developed. On the basis of data obtained from testing the process of operational use within a real transport system, values of p_{ij} probabilities and λ_{ij} intensities of the process state changes were estimated. Next, boundary probability values p_i^* were determined. These values are presented in Tab. 1 and 2.

Data on operational use concerns 119 technical objects (city buses) used in a given real system of city bus transport, in the period from 01.2005 to 09.2006. The tests were carried out by the method of passive experiment, in conditions natural for operation of the tested objects.

Tab. 1. Values of boundary probabilities p_i^* for Markov's chain

p_1^*	p_2^*	p_3^*	p_4^*
0.1629	0.0138	0.1977	0.0362
p_5^*	p_6^*	p_7^*	p_8^*
0.0356	0.0200	0.0148	0.0078
p_9^*	p_{10}^*	p_{11}^*	p_{12}^*
0.0015	0.1629	0.1889	0.1150
p_{13}^*	p_{14}^*	p_{15}^*	p_{16}^*
0.0026	0.0073	0.0269	0.0061

Tab. 2. Values of boundary probabilities p_i^* for Markov's process

p_1^*	p_2^*	p_3^*	p_4^*
0.3180	0.0026	0.5227	0.0013
p_5^*	p_6^*	p_7^*	p_8^*
0.0031	0,0015	0.0016	0.0020
p_9^*	p_{10}^*	p_{11}^*	p_{12}^*
0.0002	0.0998	0.0062	0.0048
p_{13}^*	p_{14}^*	p_{15}^*	p_{16}^*
0.0034	0.0010	0.0308	0.0009

4. Conclusions

On the basis of determined boundary probabilities p_i^* for Markov's chain, it can be said that the highest probability of entering the process states, applies to the states:

- S_3 task realization during the ride,
- S_{11} refueling,
- S_1 waiting for the task realization on the hardstand of the depot,
- S_{10} waiting for entering the serviceability assurance subsystem,
- S_{12} waiting for servicing on the day of operation.

High value of p_{11}^* probability results from the fact that in some facilities the vehicles are refueled two times during 24 hours due to the scale of the task to be accomplished or necessity to refuel after a repair done.

From the obtained probability value p_4^* it results that above 18% of technical objects which realize their tasks are damaged during their rides. It can also be said that most of them (98.2%) are diagnosed and repaired (95.9%) by the road service units. Repairs done by the road service are usually minor ones (56.2%), including control or replacement of small elements, and they do not involve lost of a ride. The other repairs (43.8%) are carried out during time longer than the break between rides, which involves the necessity of replacing them with substitute objects. On the basis of probability value p_{15}^* it results that more than 16.5% of the vehicles carrying out transport tasks are repaired in the Servicing Station. It can also be said that only part of them is diagnosed before the repair (almost 27%), and after it (more than 22.7%).

Basing on probability values p_i^* determined for Markov's process, it can be concluded that states S_3 (carrying out the task), S_1 (waiting for serviceability assurance on the hard stand) – total 84% of the operational use time, and S_{10} (waiting for the entrance to the serviceability assurance subsystem) almost 10% of the whole operational use time, are the states in which the statistical technical object stays for the longest time. The remaining time of operational use is devoted to the processes of the technical object serviceability assurance (6% of the operational use time), though being of big importance from the point of view of the transport task accomplishment, and costs born in the course of realization of these processes. In the process of serviceability assurance is 65.5%, servicing 17.5%, refueling 13%, diagnosing 4%. The summary share time of the serviceability assurance for technical objects by technical service units (p_5^*+p_6^*+p_7^*) is only slightly higher than 0.6% of the whole process of operational use.

References

[1] Dynkin, E. B., Juskevic, A. A., *Controlled Markov processes*, Springer Verlag, Berlin 1979.
[2] Fleming, W. H., Soner, H. M., *Controlled Markov processes and viscosity solutions*, Springer Verlag, New York 1993.
[3] Grabski, F., Jaźwiński, J., *Funkcje o losowych argumentach w zagadnieniach niezawodności, bezpieczeństwa i logistyki*, WKiŁ, Warszawa 2009.
[4] Iosifescu, M., *Skończone procesy Markowa i ich zastosowanie*, PWN, Warszawa 1988.
[5] Jaźwiński, J., Grabski, F., *Niektóre problemy modelowania systemów transportowych*, Instytut Technologii Eksploatacji, Warszawa-Radom, 2003.
[6] Kulkarni, V. G., *Modelling and analysis of stochastic systems*, Chapman & Hall, New York 1995.
[7] Kowalenko, I. N., Kuzniecow, N. J., Szurienkow, W., M., *Procesy stochastyczne. Poradnik*, PWN, Warszawa 1989.
[8] Leszczyński, J., *Modelowanie systemów i procesów transportowych*, Wydawnictwo Politechniki Warszawskiej, Warszawa 1994.
[9] Woropay, M., Migawa, K., *Markov model of the operational use process in an autonomous system*, Polish Journal of Environmental Studies, Vol. 16, No. 4A, 2007.
[10] Woropay, M., Migawa, K., *Markov model of the serviceability assurance process within an autonomous system*, Archiwum Motoryzacji, Nr 4, 2007.